

Analysis of Variance (ANOVA)

One-Way ANOVA

- A method for comparing several means along a single variable
 - It is the same as an independent samples t-test, but for 3 or more samples
- Called “one way” when it looks at a single variable
- Uses variance between groups compared to variance within groups to determine whether or not samples significantly differ
 - *Between groups*: takes weighted (grand) mean of all samples and looks at how far each sample mean is from the grand mean
 - *Within group*: takes variance of each sample, weights by sample size and then sums all variances for all samples

Derives the equation

$$F_{i-1, n-i} = \frac{MS_b}{MS_w}$$

- Relies on the F-distribution to assess probability and significance

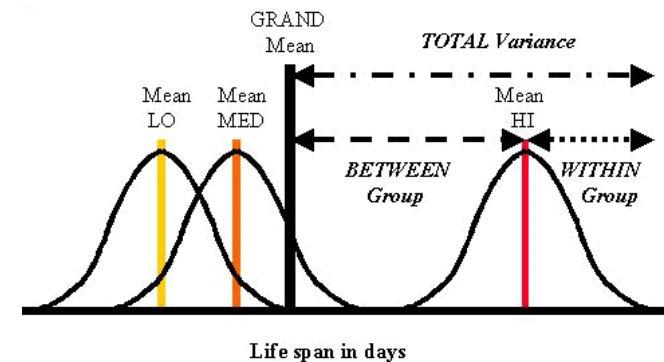
Assumptions of ANOVA

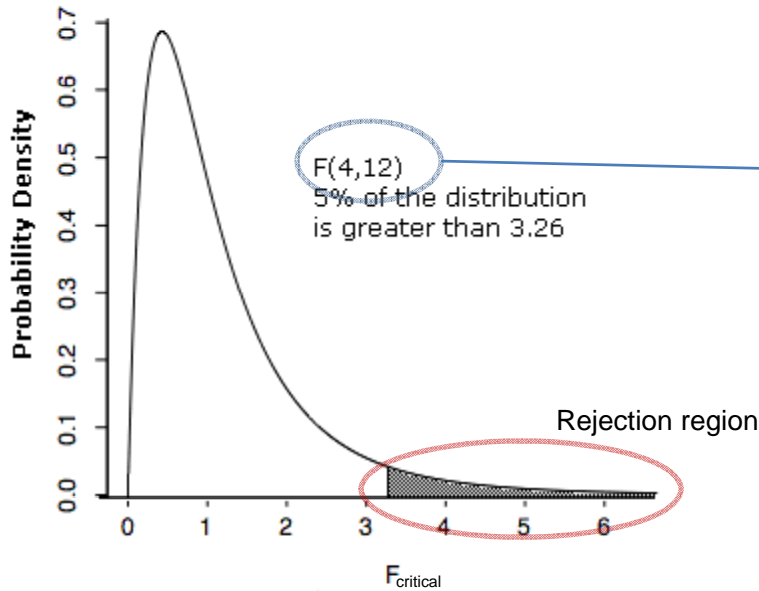
- 1) Samples are independent
- 2) The variable is normally distributed

- Hypotheses

- $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 \dots = \mu_k$

- H_a : Not all of the means are equal





$$df = i - 1, n - i$$

= df, 5 samples compared & total n = 17

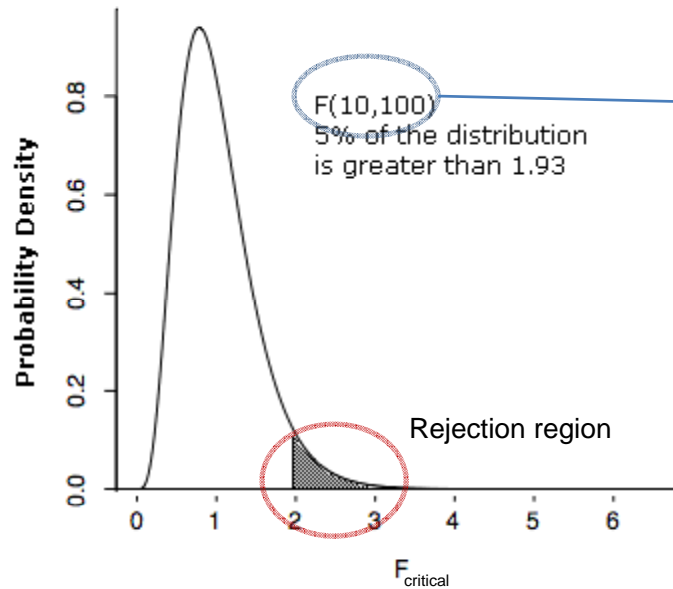
F-critical = 3.26

i = number of samples

n = total of all sample sizes

i = 5, i - 1 = 4

n = 17, n - i = 12



= df, 11 samples compared & total n = 111

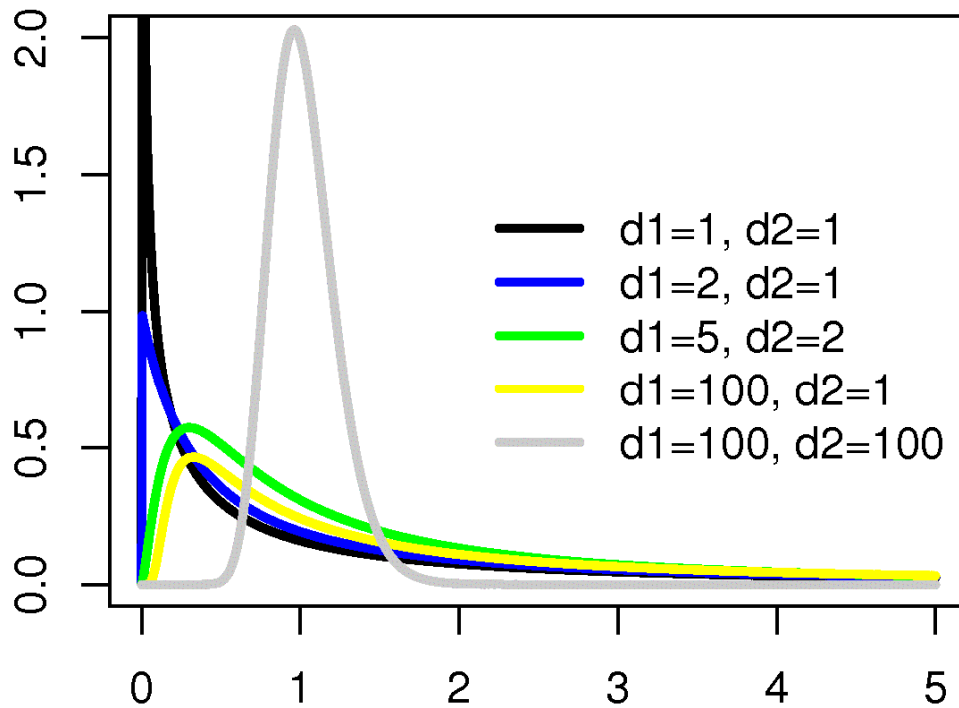
F-critical is 1.93

$$F_{i-1, n-i} = \frac{MS_b}{MS_w}$$



F = between group variance / within group variance

The F-distribution



Flatter, longer right tail when sample sizes are small

Makes it harder to reach F -critical with the test F and harder to yield significant results

That is, the longer tail stretches the rejection region out to the right

For larger samples, the rejection region is compressed to the left and a lower test F surpasses F -critical

What you need to know...

- If ANOVA F is $> F$ -critical then you reject the H_0
- If $p < \alpha$ then your ANOVA F is $> F$ -critical
- Remember α you set; p is the probability of type I error for the test (ANOVA F)

Inextricably intertwined and intertwined

That is, they (“critical vs. test” & “ α vs. test p ”) tell you the **SAME THING!**
The same with t , F , U , H , R , X , and all other test statistics, everywhere and forever....

Phrasing results

- No significant difference was found [$F(df, df) = ???, p = ????$], mean 1 was ??? ($s = ????$), mean 2 was ...
= all of the means are the same, accept H_0
- A significant difference was found [$F(df, df) = ???, p = ????$], mean 1 was ??? ($s = ????$), mean 2 was ...
= at least one mean is different, reject H_0

Look familiar???

- If t is $> t$ -critical then you reject the H_0
- If $p < \alpha$ then your t is $> t$ -critical
- Remember α you set; p is the probability of type I error for the test (Student's t)

What if normality cannot be assumed?



- $n < 30$
- AND you do not know if the population is normally distributed
- Then you should not assume normality



- *What to do? Keep calm...*
- Convert to ordinal scale/position data and run non-parametric tests
 - For example, test difference in medians instead of means



Stay cool...



- There are non-parametric versions of all of the tests we have learned or will learn
- Mann-Whitney U test (Wilcoxon Rank Sum)
 - H_0 : Median₁ = Median₂
 - H_a : Median₁ \neq Median₂
- This test is uses mean ranks

Mann Whitney U

Sample 1 Tinted	Sample 2 Untinted
\$175	\$170
\$165	\$215
\$200	\$190
\$185	\$240
\$155	\$215
\$145	\$210
\$170	\$175
	\$255
	\$220

Sorted Electricity Cost	Cost Rank	Sample 1 Rank	Sample 2 Rank
145	1	1	
155	2	2	
165	3	3	
170	4.5	4.5	
170	4.5		4.5
175	6.5	6.5	
175	6.5		6.5
185	8	8	
190	9		9
200	10	10	
210	11		11
215	12.5		12.5
215	12.5		12.5
220	14		14
240	15		15
255	16		16

Mann Whitney U SPSS Output

	<u>N</u>	<u>Mean Rank</u>	<u>Sum of Ranks</u>
Tinted	7	5.00	35.00
Untinted	9	11.22	101.00
Total	16		

Test Statistics	
Mann-Whitney U	7.000
Z	-2.599
Asymp. Sig. (2-tailed)	.009

$$U = T_1 - \frac{n_1(n_1 + 1)}{2} = 35 - \frac{7(7 + 1)}{2} = 35 - 28 = 7.$$

Actual sum of the ranks

Minimum sum of the ranks

Kruskal-Wallis H Test

- Non-parametric One Way ANOVA
 - 3 or more independent samples
 - H_0 : Median₁ = Median₂ = Median₃ = ...Median_k
 - H_a : Not all of the medians are equal
- This test is uses mean ranks

Kruska-Wallis H SPSS Output

<u>Sample 1</u> <u>Tinted</u>	<u>Sample 2</u> <u>Untinted</u>	<u>Sample 3</u> <u>Untinted,</u> <u>single paned</u>
\$175	\$170	\$220
\$165	\$215	\$365
\$200	\$190	\$185
\$185	\$240	\$245
\$155	\$215	\$195
\$145	\$210	\$298
\$170	\$175	\$425
	\$255	
	\$220	

	<u>N</u>	<u>Mean Rank</u>
Tinted	7	5.36
Untinted	9	13.06
Sing.Untint.	7	17.29
Total	23	
Test Statistics		
Chi-Square (H)		11.212
Df		2
Asymp. Sig.		.004

What do you need to know...?

- That Mann-Whitney U (aka Wilcoxon Rank Sum) is a non-parametric cousin to the t test
- That Kruskal-Wallis H is a non-parametric cousin to One-Way Anova
- You need to know the H_0 and H_a for Mann Whitney and Kruskal Wallis
- That each has a test-statistic (U for Mann Whitney; H for Kruskal-Wallis)
- That p for U or H is interpreted the same way as p for t or F in terms of rejecting H_0 (see inextricable intertwining from earlier slide)

ANOVA & K-W in SPSS

The screenshot shows the SPSS Data Editor interface with the 'Analyze' menu open. The 'Compare Means' option is selected, and the 'One-Way ANOVA...' dialog box is displayed. The dialog box shows the following configuration:

- Dependent List: AllTeeth
- Factor: GroupALL

The background data table is as follows:

	Midwest	Missouri
1	21.08	24.4
2	22.74	24.4
3	22.90	25.0
4	22.98	26.3
5	23.17	27.4
6	23.86	27.5
7	24.61	27.6
8	25.00	27.7
9	25.22	27.8
10	25.30	27.9
11	25.46	28.2
12	25.89	28.3
13	26.02	28.3
14	26.14	28.30
15	26.27	28.40

Cont'd

Options

Post hoc

The screenshot shows the 'One-Way ANOVA: Options' dialog box. The 'Dependent List' contains 'AllTeeth'. The 'Statistics' section has 'Descriptive' checked, and 'Means plot' is unchecked. The 'Missing Values' section has 'Exclude cases analysis by analysis' selected. The background shows a list of groups (Midwest, Missouri, Pleist, GroupMM, MidMiss, GroupMisF, MissPleist) and a data table with values like 26.14, 26.22, 26.34, 26.42, 26.42, 26.47.

26.14				1.00	
26.22				1.00	
26.34				1.00	
26.42				1.00	
26.42				1.00	
26.47	29.40	31.40	1.00	26.47	1.00

The screenshot shows the 'One-Way ANOVA: Post Hoc Multiple Comparisons' dialog box. Under 'Equal Variances Assumed', 'Tukey' is checked. Under 'Equal Variances Not Assumed', 'Tamhane's T2', 'Dunnnett's T3', 'Games-Howell', and 'Dunnnett's C' are all unchecked. The 'Significance level' is set to .05. The background shows a data table with values like 21.08, 22.74, 26.42, 26.47.

21.08					
22.74					
26.42					
26.47	29.40	31.40	1.00	26.47	1.00

Output

Descriptives

AllTeeth

	N	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean		Minimum	Maximum
					Lower Bound	Upper Bound		
1.00	32	26.1378	2.09714	.37073	25.3817	26.8939	21.08	29.78
2.00	21	27.7476	1.53773	.33556	27.0477	28.4476	24.40	29.70
3.00	20	29.8270	1.17467	.26266	29.2772	30.3768	27.30	31.90
Total	73	27.6116	2.29099	.26814	27.0771	28.1462	21.08	31.90

ANOVA

AllTeeth

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	168.054	2	84.027	28.029	.000
Within Groups	209.847	70	2.998		
Total	377.901	72			

Multiple Comparisons

Dependent Variable: AllTeeth

Tukey HSD

(I) GroupALL	(J) GroupALL	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1.00	2.00	-1.60981*	.48625	.004	-2.7742	-.4455
	3.00	-3.68919*	.49353	.000	-4.8710	-2.5074
2.00	1.00	1.60981*	.48625	.004	.4455	2.7742
	3.00	-2.07938*	.54097	.001	-3.3748	-.7840
3.00	1.00	3.68919*	.49353	.000	2.5074	4.8710
	2.00	2.07938*	.54097	.001	.7840	3.3748

*. The mean difference is significant at the .05 level.

Kruskal Wallis

Bear Data.sav [DataSet1] - SPSS Data Editor

File Edit View Data Transform Analyze Graphs Utilities Window Help

1 : Midwest

	Midwest	Missouri	AllTeeth	GroupMM	MidMiss
1	21.08	24.4	21.08	1.00	21.0
2	22.74	24.4	22.74	1.00	22.7
3	22.90	25.0	22.90	1.00	22.9
4	22.98	26.3	22.98	1.00	22.9
5	23.17	27.4	23.17	1.00	23.1
6	23.86	27.5	23.86	1.00	23.8
7	24.61	27.6	24.61	1.00	24.6
8	25.00	27.7	25.00	1.00	25.0
9	25.22	27.8	25.22	1.00	25.2
10	25.30	27.9	25.30	1.00	25.3
11	25.46	20.2	25.46	1.00	25.4
12	25.89	28.3	25.89	1.00	25.8
13	26.02	28.3	26.02	1.00	26.0
14	26.14	28.30	26.14	1.00	26.1
15	26.22	28.40	26.22	1.00	26.2
16	26.34	28.70	26.34	1.00	26.3
17	26.42	28.80	26.42	1.00	26.4

Analyze > Nonparametric Tests > K Independent Samples...

Tests for Several Independent Samples

Test Variable List: AllTeeth

Grouping Variable: GroupALL(? ?)

Test Type: Kruskal-Wallis H Median

Options...

Tests for Several Independent Samples

Several Independent Samples: Define Range

Range for Grouping Variable

Minimum: 1

Maximum: 3

Test Type: Kruskal-Wallis H Median

Options...

Descriptive Statistics

	N	Mean	Std. Deviation	Minimum	Maximum
AllTeeth	73	27.6116	2.29099	21.08	31.90
GroupALL	73	1.8356	.83356	1.00	3.00

Kruskal-Wallis Test

Ranks

	GroupALL	N	Mean Rank
AllTeeth	1.00	32	23.09
	2.00	21	36.98
	3.00	20	59.28
Total		73	

Test Statistics^{a,b}

	AllTeeth
Chi-Square	35.811
df	2
Asymp. Sig.	.000

a. Kruskal Wallis Test

b. Grouping Variable: GroupALL